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National Committee for Fluid Mechanics Films

FILM NOTES

for

RHEOLOGICAL BEHAVIOR OF FLUIDS*

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Introduction

Rheology in its broadest sense is the study of the relation between stress and deformation in continuous media. For the *inviscid fluid* model this relation is simply that the *shear stresses are always zero* — or equivalently that the stress is always isotropic. In the *Newtonian fluid* model, the stress is isotropic only when the fluid is stationary; when it is in relative motion, the *stresses are assumed to be linear functions of the instantaneous velocity gradients.* The purpose of the film is to show examples of fluid behavior which cannot be approximated by either the inviscid fluid or Newtonian fluid models.

Yield Value

There are materials, particularly suspensions, which flow only when the shear stress exceeds a critical value, known as a yield value. The clay suspension in Fig. 1, for example, does not flow through the tube either under its own weight or with the driving pressure of a small added weight. Only when a sufficiently large weight is added does it flow. If the large weight is removed, the flow immediately stops. Such substances can support a shear stress in a state of equilibrium if the shear stress is less than a critical yield value.

The Memory Fluid Model

Some materials, such as molten plastics, protein solutions (e.g., egg white), and rubber cement, flow under the slightest shear stress, but are still not Newtonian. For these the *memory fluid* model provides a good description. In this model, the stress is isotropic only when the fluid has been stationary for a long time; otherwise *the stress is assumed to be a non-linear function of the deformation-gradient history.*

It is instructive to consider the effects of non-linearity and history independently, although they are really not separable. The dependence on history affects the non-linear phenomena even in steady flow, and there are non-linearity effects with large deformation in the memory experiments.

Dependence on History

The current value of the stress in a memory fluid cannot be determined from a knowledge of the current state of deformation alone, since it depends on the

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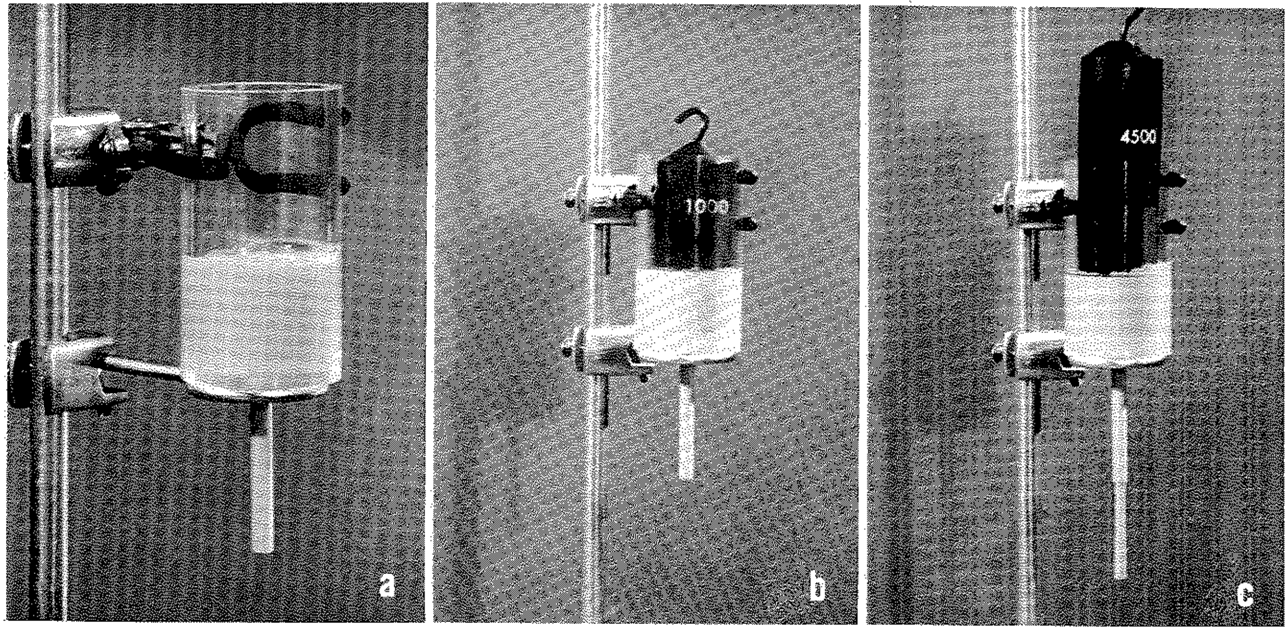
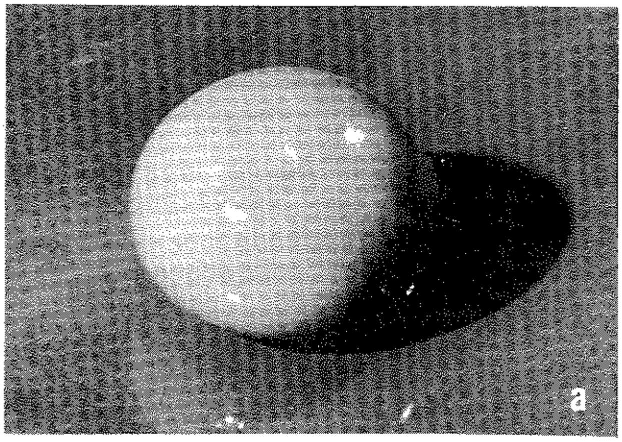
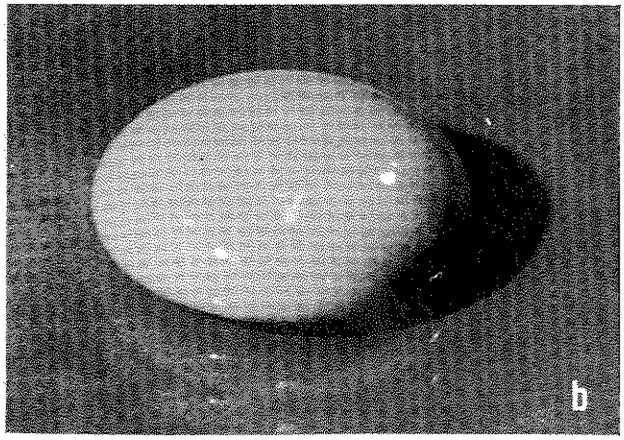


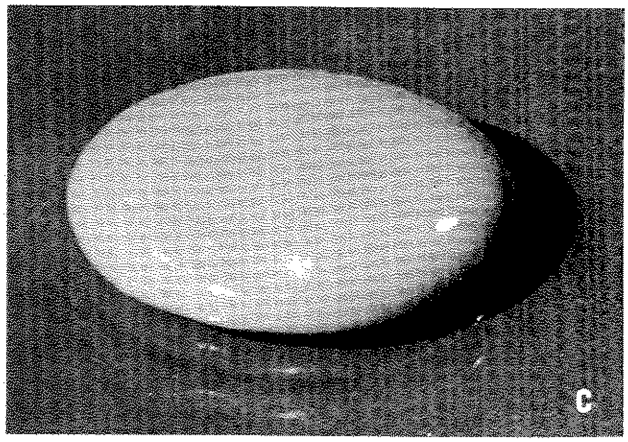
1. A clay suspension in a cylinder does not flow out through the open exit tube under its own weight (a) or even when a small weight is placed on the light piston (b). With a larger weight the suspension does flow (c). The shear stress must exceed a critical yield value for flow to occur.

deformation which the material has experienced previously. Conversely, the current stress does not determine the current state of deformation. As a consequence, memory fluids show elastic-like behavior (hence the term *viscoelastic* is frequently used for these fluids). However, unlike the elastic solid, the memory fluid does not have a preferred configuration to which it invariably returns when all stresses are removed.

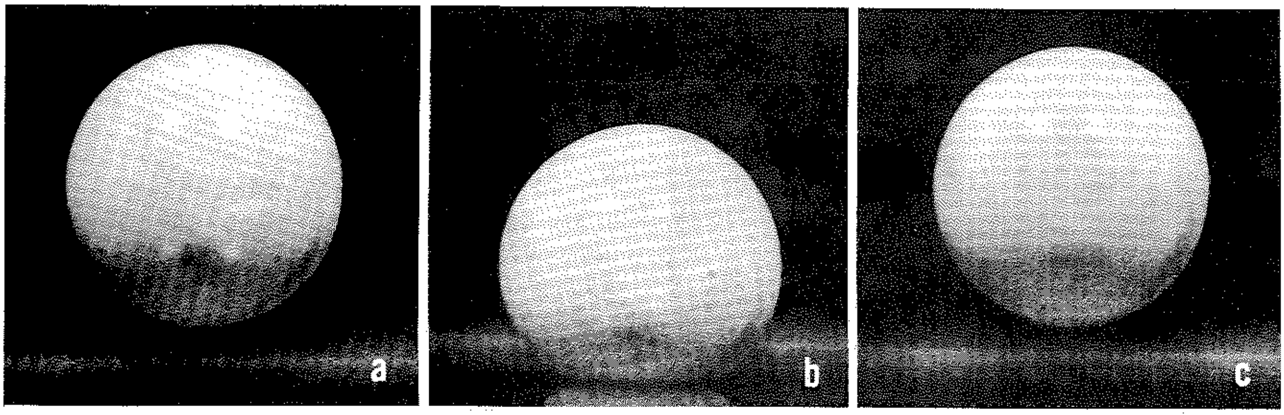
In Fig. 2 the silicone-putty ball flows into a puddle even under such a small force as that of gravity, if given sufficient time. But during the short time of a rapid impact (Fig. 3), it behaves like an elastic solid because it does not have a chance to "forget" its previous spherical shape. The significant relaxation times for stresses in silicone putty are much less than the time for settling due to gravity, but much greater than the time of rapid impact. Thus the putty may exhibit a range of behavior from elastic to viscous, depending on how the characteristic time of the experiment compares with the significant relaxation times.







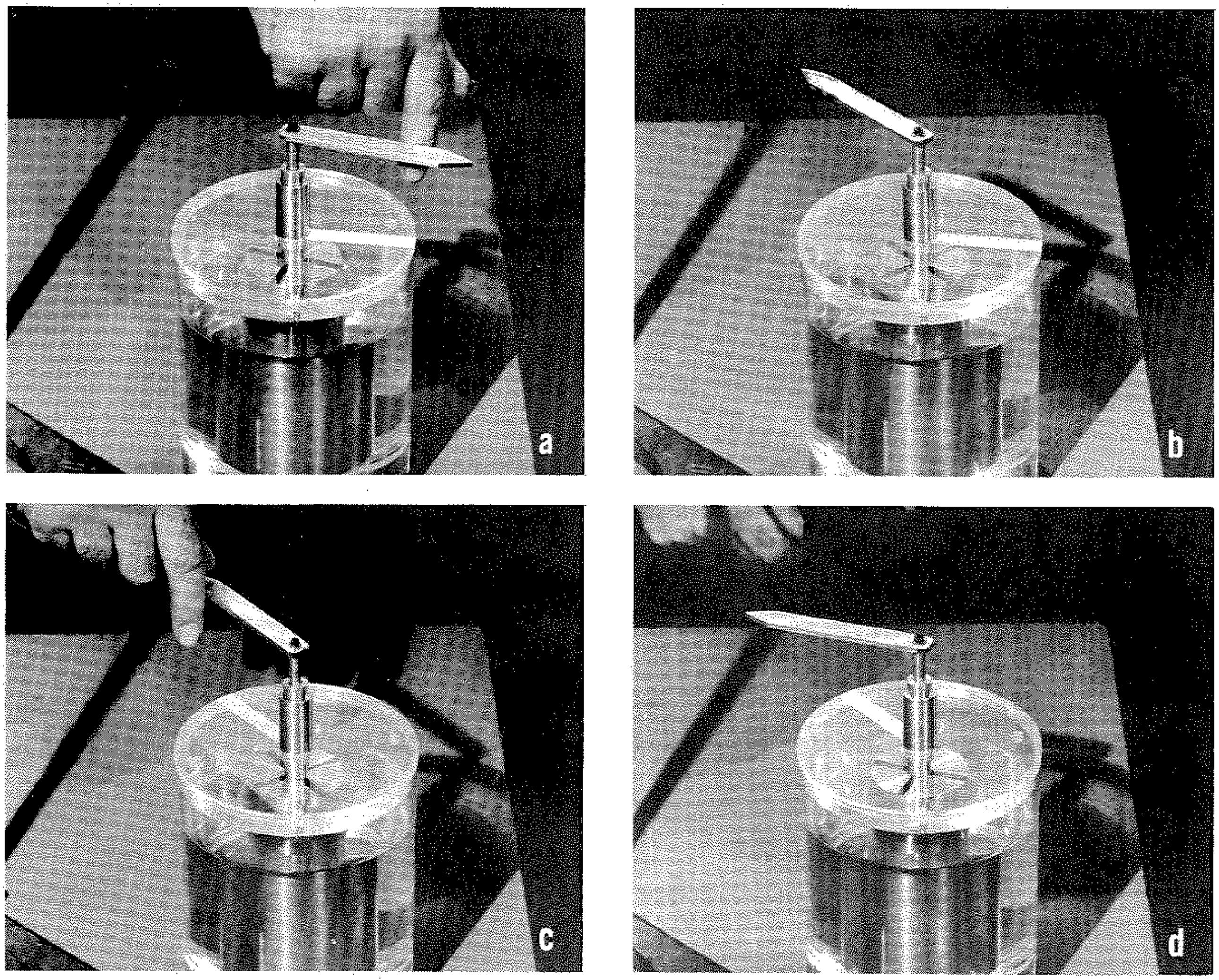
2. Ball of silicone putty relaxes into a puddle in an hour under the force of its own weight.



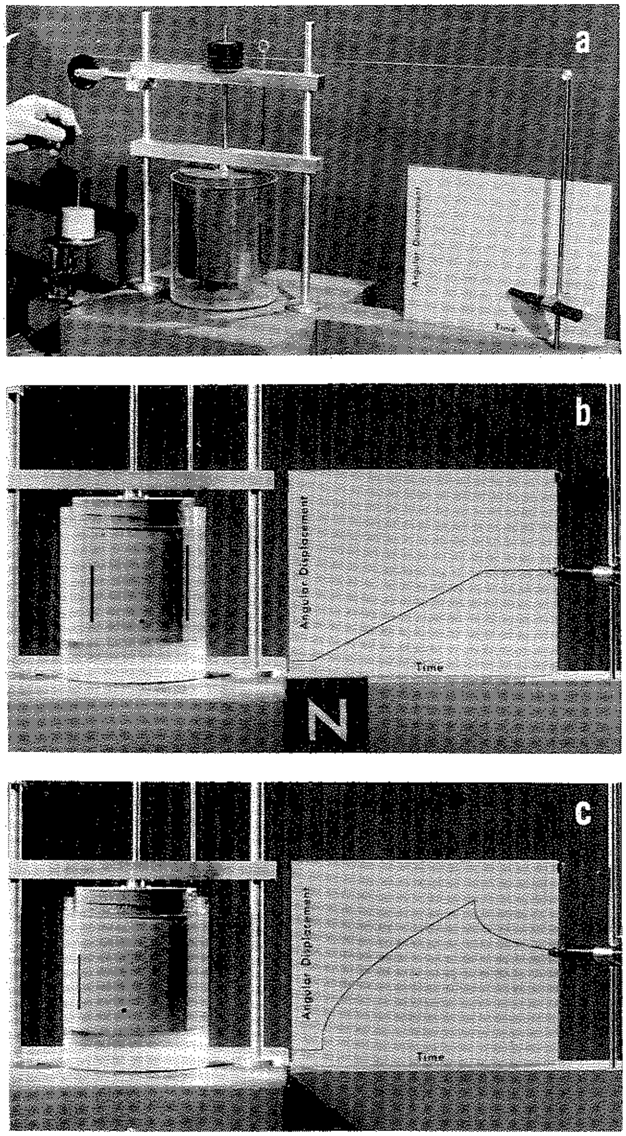
3. When bounced on a hard surface, the silicone-putty ball deforms on impact, but recovers its original shape immediately on the rebound, like an elastic solid.

A similar phenomenon can be observed when a shear is applied to a polymer solution between coaxial cylinders (Fig. 4a). When the inner cylinder is quickly rotated and then immediately released, almost half the deformation is recovered (Fig. 4b). When the inner cylinder is held for a while before being released, the deformation recovered is much less (Fig. 4d), because of the fading memory of the fluid.

The deformation history can be followed more precisely in the coaxial cylinder apparatus of Fig. 5. A constant torque is applied to the inner cylinder, and



4. A polymer solution is in the annulus between coaxial cylinders. The white stripe on the top marks the starting positions. When the inner cylinder is quickly rotated clockwise through 360° (a) and released immediately, the fluid pulls it back almost half the way to its original position (b). When, instead, the inner cylinder is rotated clockwise but held for some time in a 360° position (c), the deformation recovery upon release is much less (d).



5. **(a) The fluid in the annulus between coaxial cylinders is stressed by suddenly subjecting the rotatable inner cylinder to a constant torque through the pulley-and-weight system. The torque is removed when the weight reaches the platform and disengages from the string by means of a slip toggle. The pen is moved vertically by a string attached to the driving drum on the inner cylinder. With the chart moving at uniform speed to the left, angular position of the inner cylinder vs. time is recorded. (b) Record for a Newtonian fluid ("N"). (c) Record for a polymer solution.**

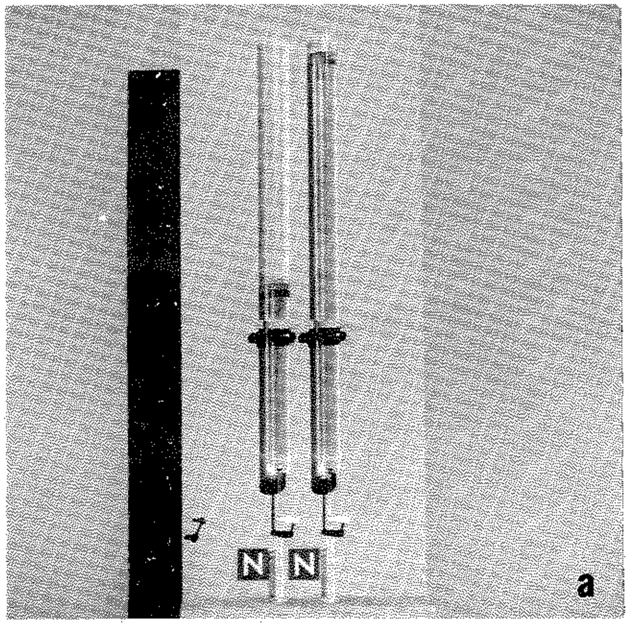
later removed. Its angular rotation as a function of time is recorded on the chart. With the Newtonian fluid (Fig. 5b), a constant speed of rotation is obtained almost as soon as the torque is applied, and the motion ceases when the weight is removed. With the polymer solution in the apparatus (Fig. 5c), it takes considerable time for the inner cylinder to achieve a constant rate of rotation. It is clear that the motion at any instant is not determined simply by the value of the instantaneous stress; although the torque is constant, the rate of rotation is changing. Furthermore, when the torque is removed, the material deforms although no stress is being applied. The reversal in direction also indicates that not all the energy used to produce the flow was dissipated; some was stored and then recovered.

Non-linearity and Shear Stresses

The non-linearity in the stress-deformation relation of the memory fluid affects both the shear stresses and the normal stresses in simple steady flow situations.

The effect on shear stresses is seen in experiments of the type usually used to measure viscosity. For the Newtonian fluid, when the pressure head driving the fluid through a tube is doubled (Fig. 6a), the rate of flow is also doubled (Fig. 6b). With some fluids, such as some polymer solutions, doubling the head more than doubles the rate of flow (Fig. 6c). This behavior is called *pseudoplastic* or *shear-thinning*. With some suspensions, a contrary situation arises (Fig. 6d). This is called *dilatant* or *shear-thickening* behavior.

To predict the flow behavior of an incompressible Newtonian fluid, only the density and the viscosity are needed. For an incompressible memory fluid, the rate of flow through tubes in steady laminar flow is governed by the density and a *viscosity function.** This function also determines the velocity profile, which is parabolic for the Newtonian fluid (Fig. 7a), but can be quite different for a more complex fluid (Fig. 7b).



6. **Non-linearity in pipe flow. The same fluid is contained in two identical large-diameter reservoirs which are provided with identical outlet tubes and flapper valves which open and close simultaneously. The heads (measured from the bottom of the tubes to the free surfaces) are in the ratio of two to one, and remain nearly so during the experiment. The experiment is performed with three different fluids in succession.**

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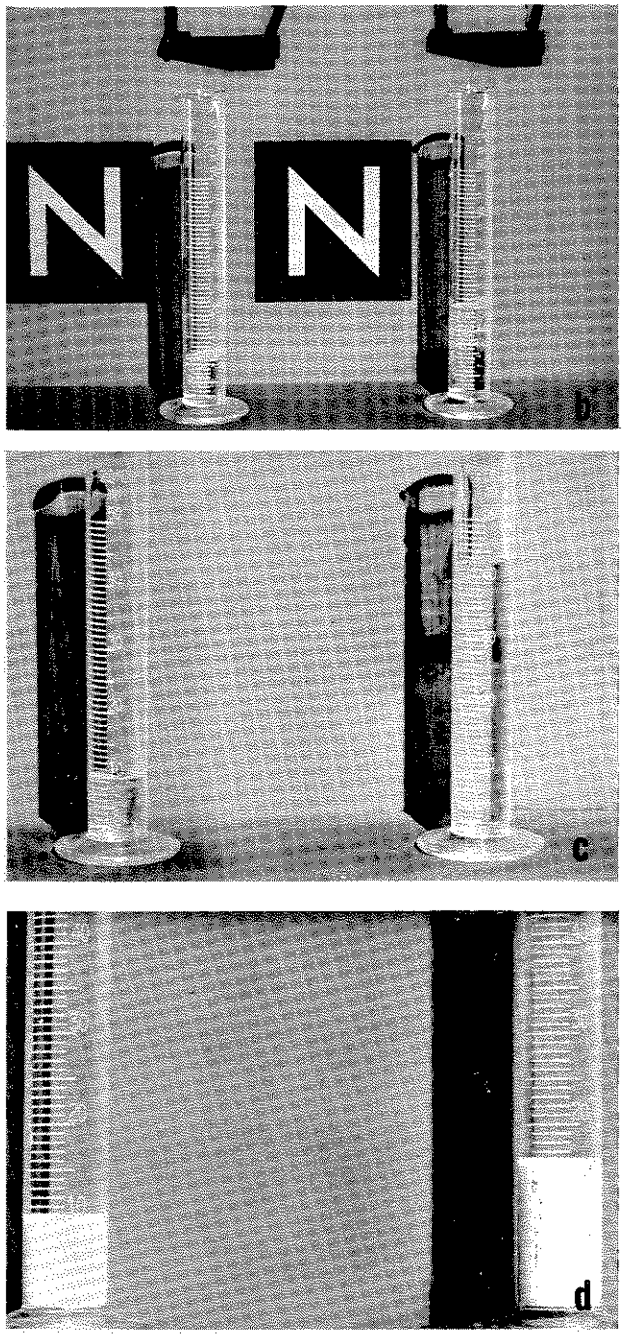
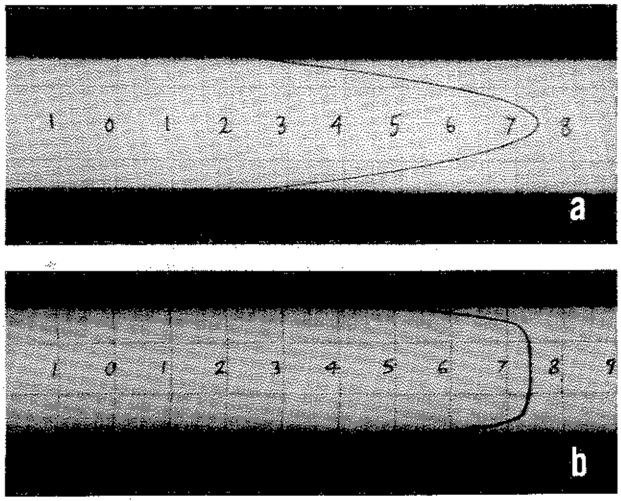
*The viscosity function also governs the relation between torque and angular velocity in steady laminar shearing between rotating coaxial cylinders, and in some other viscometers.

In *shear-thinning* behavior the viscosity function has lower values at higher rates of shear. In *shear-thickening* behavior, the opposite holds (Fig. 8).

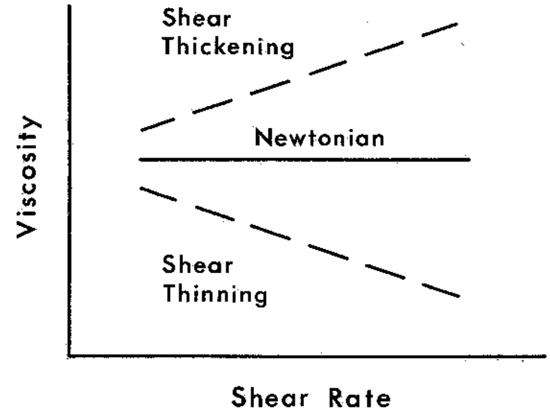
In the "race experiment" illustrated in Fig. 9, the flow rate under a varying head of a fluid with shear-thinning behavior was compared to that of a Newtonian fluid. The conditions were deliberately selected so that at the higher rates of shear associated with the initially larger pressure head the viscosity of the polymer solution was much lower than that of the Newtonian fluid. Near the end of the experiment, at the low rates of shear associated with the low pressure head, the value of the viscosity for the polymer solution was much greater. Thus, although the shear-thinning fluid achieved an early lead in the race to empty the reservoirs, it was overtaken near the end by the Newtonian fluid.



(Fig. 6 continued) With the Newtonian fluid of (b), the flow rates are linearly related to the driving heads. With the polymer solution of (c), doubling the head more than doubles the flow rate (shear-thinning behavior). With the suspension of (d), the rate of flow at twice the head is less than doubled (shear-thickening behavior).



7. The position of an initially transverse straight line of marked fluid in a pipe of circular cross-section is shown shortly after impulsive start of flow. With the viscous Newtonian fluid of (a), the deformation of the line indicates a parabolic velocity profile. With the polymer solution of (b), the indicated velocity profile is not parabolic.* (Courtesy A. G. Fredrickson and N. N. Kapoor, University of Minnesota)



8. Viscosity functions.

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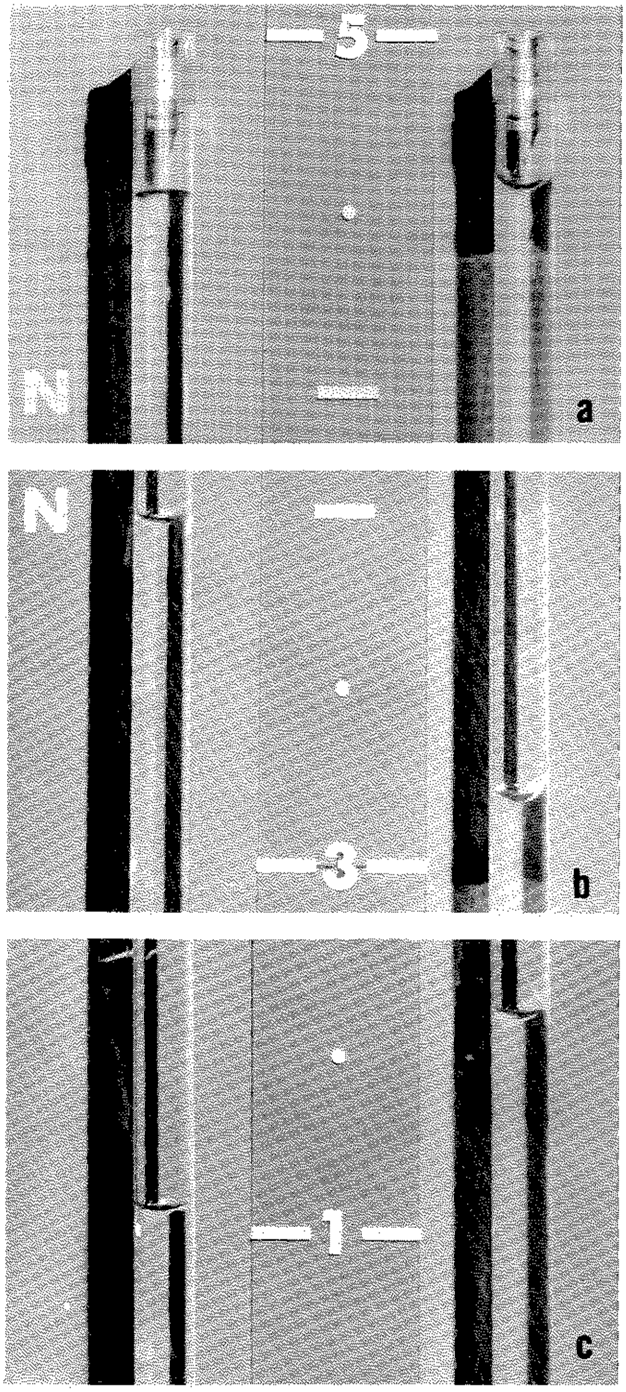
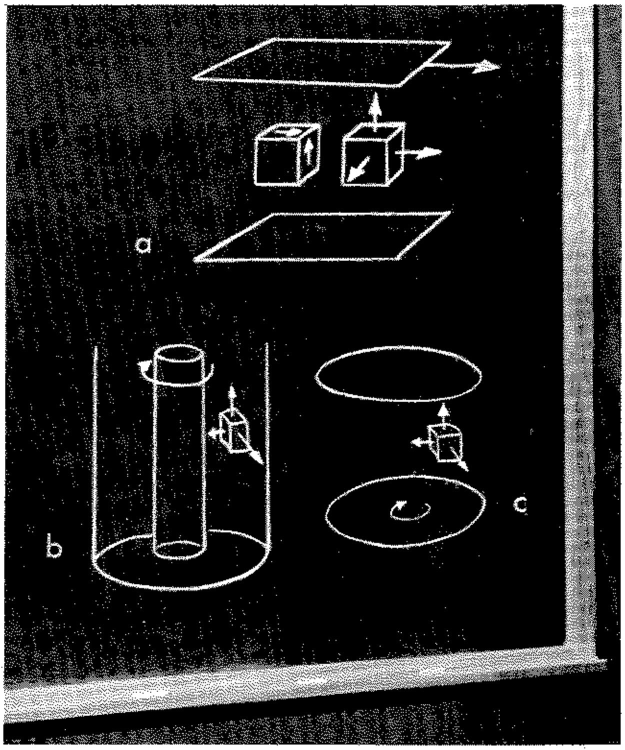
*The marked line of fluid of Fig. 7b retains approximately the same shape as the flow proceeds, with most of the velocity gradient concentrated near the walls of the tube. This indicates that non-linearity in the viscosity function is responsible for the shape of the profile and that it is not simply an early stage in the development of a parabolic profile.



9. The race experiment. (a) Identical reservoirs with identical outlet tubes are filled to the same level with a Newtonian fluid (left) and a "shear-thinning" fluid (right). The flapper valves are opened simultaneously. At first, the non-Newtonian fluid leads in the race to empty (b), but as the heads get smaller, the non-Newtonian fluid slows down relative to the Newtonian, which ultimately wins the race (c).

Non-linearity and Normal Stresses

Non-linearity in the stress-deformation relation affects not only the shear stresses but also the normal stresses. In steady simple shearing flow between infinite parallel plates, the normal stresses on an infinitesimal volume element in the three directions indicated in Fig. 10a are all equal for the Newtonian fluid, but not for the memory fluid. The same is true for the normal stresses in the flow between rotating cylinders (Fig. 10b), in the flow between rotating parallel discs (Fig. 10c), and in some other simple laminar flows. As a result, the *distribution* of normal stresses can be quite different for a memory fluid from what it is for the Newtonian fluid.



10. Normal stresses on volume element in (a) simple shear, (b) circular Couette flow, (c) torsional shear.

For example, in the flow of a Newtonian fluid between rotating cylinders (Fig. 11a), a higher normal stress is exerted on the outer cylinder wall than on the inner wall. For the polymer solution just the opposite is observed* (Fig. 11b). The shape of the surface of the fluid in the annulus is also quite different in the two cases.

The normal stress distribution is also quite different for Newtonian and memory fluids in the flow between

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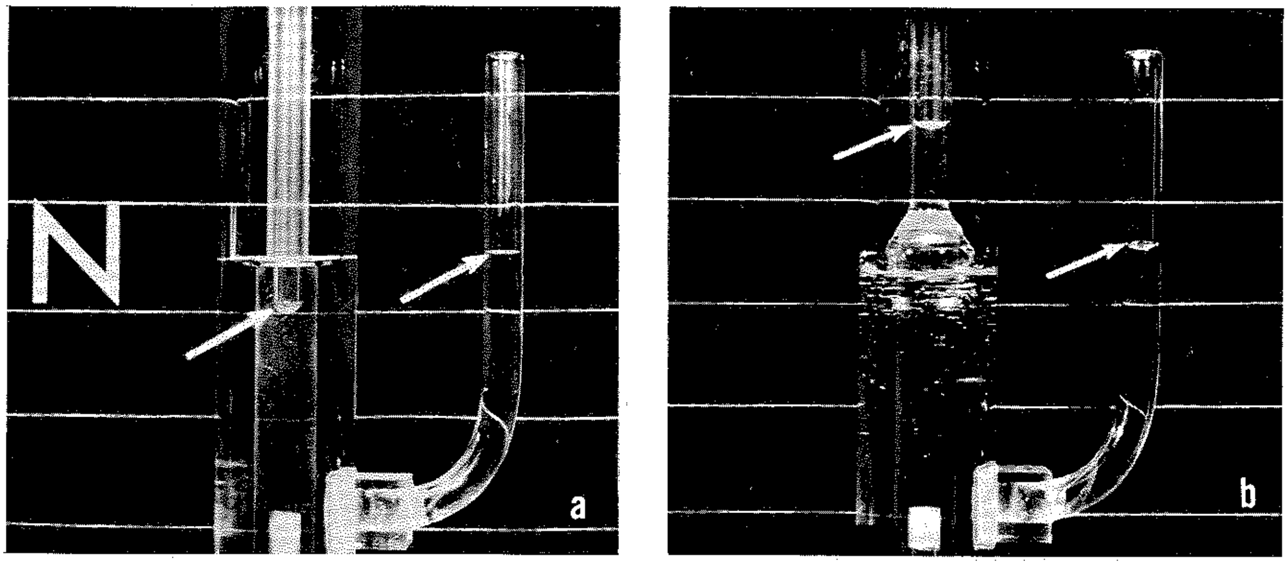
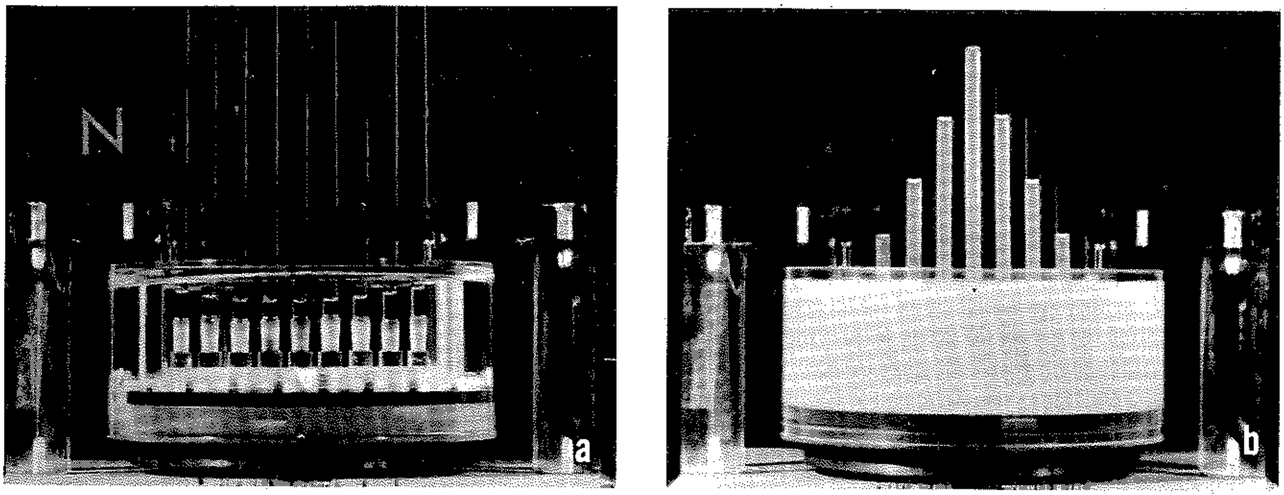
*This experiment is sufficient to indicate the inapplicability of the theory of the viscoinelastic fluid (sometimes called the Reiner-Rivlin fluid) whose force-deformation relation is based on the assumption that the fluid stress is a non-linear function of the velocity gradients. Although such a fluid exhibits normal stress effects in other flows, it would not in this experiment.

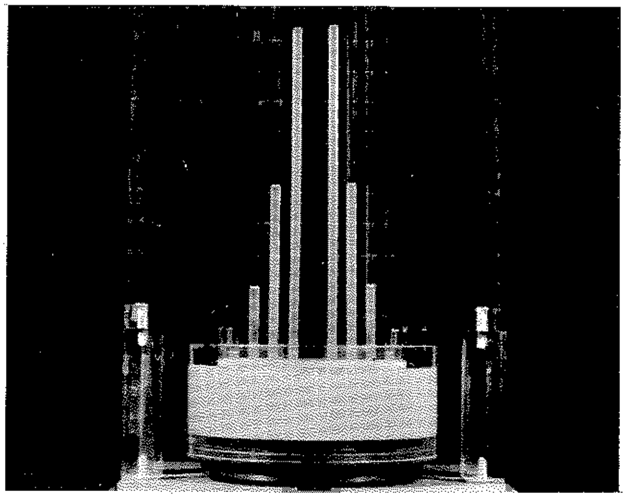


11. **Couette flow in the annulus between an outer cylinder and a coaxial rotating shaft. The shaft is a hollow tube, with a hole communicating to the annulus; the height of fluid in the tube indicates the normal stress at the hole. There is a corresponding hole and a side-arm manometer tube to indicate the stress normal to the outer cylinder. Arrows point to the liquid levels in the manometers. With the Newtonian fluid of (a), the normal stress is greater at the greater radius, owing to centrifugal forces. With the polymer solution (b), the non-linear effects overwhelm the centrifugal pressure gradient; the normal stress is substantially greater at the inner cylinder. The polymer solution also climbs the rotating shaft.**



12. **Flow between rotating parallel discs. The upper disc is stationary, and is instrumented with pressure taps and manometers across a diameter. The outer cylinder rotates, so that fluid is sheared between the upper disc and the floor of the container. The distribution of normal stress on the upper disc is shown for a Newtonian fluid (a) and for a particular non-Newtonian fluid in (b).**

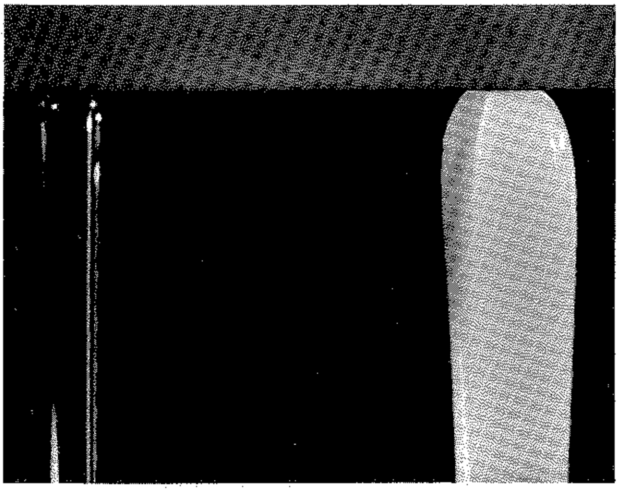
rotating parallel discs. For a Newtonian fluid the normal stress distribution is governed by centrifugal force, and the *lowest* stress occurs at the center of the plates (Fig. 12a). With the polymer solution, the *highest* normal stress is exerted at the center (Fig. 12b). A similar situation arises in the steady laminar flow between a rotating shallow cone and a flat disc (Fig. 13). For this geometry the stress normal to the stationary disc is proportional to the logarithm of the distance from the axis of rotation.



13. **Experiment similar to that of Fig. 12b except that a shallow 2-degree cone (with apex just below the center of the upper disc) constitutes the floor of the outer container. The normal stress is proportional to the logarithm of the distance from the axis of rotation.**

Other Flows

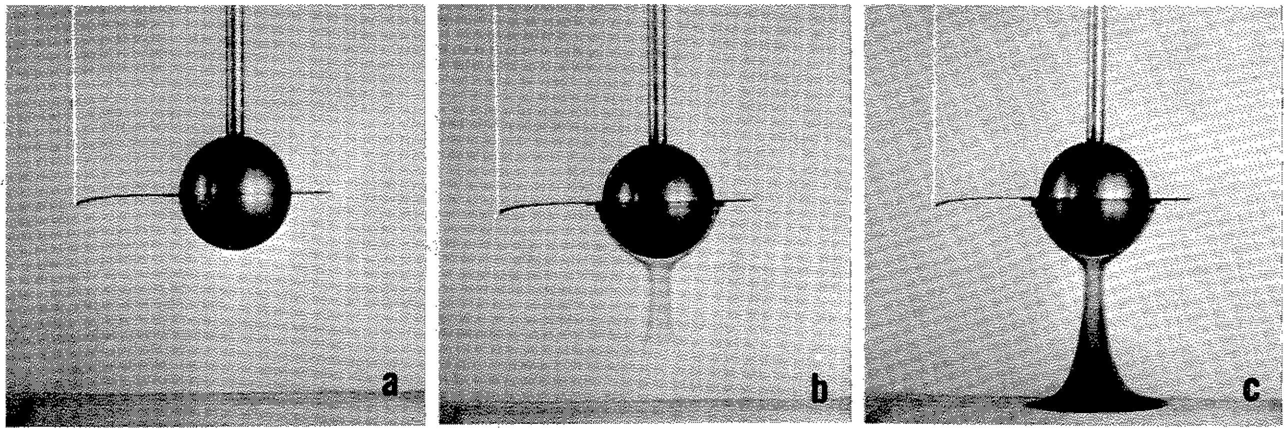
Another example where there is a dramatic difference between the behavior of Newtonian and memory fluids is in the flow through an orifice (Fig. 14). The jet of polymer solution has a considerably greater diameter than the orifice through which it has been forced, whereas the jet of glycerin (a Newtonian fluid) has a smaller diameter than the orifice. When plastic articles are to be made by the extrusion process, this effect must often be considered and the die designed smaller than the dimensions of the finished product.



14. The shapes of the jets driven by the same pressure and issuing from two identical orifices in a thick plate are compared for a Newtonian fluid (left) and a polymer solution (right).

One further example that shows sharply contrasting behavior is the flow pattern around a rotating sphere. With the sphere rotating in a high polymer solution the flow spirals inward at the equator and out at the poles (Fig. 15). Just the opposite happens for a sphere rotating in a Newtonian fluid.

15. A sphere rotating in a non-Newtonian fluid. Dye issuing from a tube at the left shows that the flow spirals radially inward in the equatorial plane and leaves the neighborhood of the sphere in the axial direction at the poles. The secondary flow induced by a sphere rotating in a Newtonian viscous fluid would be the opposite: toward the sphere at the poles and away from it at the equator. (Courtesy H. Giesekus, Farbenfabriken Bayer AG. See Reference 4.)



Conclusion

Not all non-Newtonian fluids show effects as dramatic as those chosen for demonstration in this film. However, when dealing with fluids containing long-chain polymers and suspensions, one should not be surprised to find flow patterns and stress distributions which are quite different from those predicted for a Newtonian fluid.

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